Unified Interactions Theory. Theses

A. A. Ketsaris
(Dated: September 17, 2012)

In this article we will bring together the results of a set of lectures [6] and put them together as a short introduction to the Unified Interactions Theory. Our approach is based on an algebraic generalization of two spaces: the spacetime and the action space, which is similar to the spacetime. Both the spacetime and the action space are provided properties of the tensor algebra. This allows us to explain a hierarchy of fundamental elementary particles and to make generalizations for them. The Clifford algebra as a special case of the tensor algebra is associated with leptons. Linear and bilinear transformations of the tensor algebra are associated with intermediate particles. These transformations also describe interaction between fundamental and intermediate particles.

I. INTRODUCTION

The Unified Interactions Theory is a hypothetical one intended to explain facts concerning all kinds of interactions from some universal conceptions, in a unified way. Here we propose a version of the UIT. Firstly we will enumerate the most important questions which the UIT must clarify from our standpoint. And then we will give our answers.

II. PROBLEMS OF THE UNIFIED INTERACTIONS THEORY

1. The first question is as follows. Is it possible to explain that interactions have quantum character? I. e. why do the quantities that characterize interactions get discrete values? We mean, first of all, momentum and energy. Many of the quantum theory founders considered such a statement of the problem as unproductive. They said it is enough to calculate values of discrete quantities. However it is important to clarify the problem for our understanding of a physical world picture. The problem stated generates a series of the quantum theory issues.

1.1. An operator is set in correspondence to a physical quantity. What is the meaning of such an operator? How is it constructed? For example, why is an operator

$$-i\hbar\frac{\partial}{\partial x} \qquad (1)$$

set in correspondence to the momentum p? Here i denotes the imaginary unit, $\hbar$ is the Planck constant, x is a spatial coordinate.

Let us consider the momentum

$$p = \frac{\partial S}{\partial x}\,,$$

where S is a physical quantity called *action*. Then it would be natural to associate the momentum with an operator

$$\frac{\partial}{\partial x}\,.$$

that is applied to a scalar quantity – action.

This implies some additional questions.

1.2. What does the imaginary unit that enters into the quantum operator mean? Why can not we use another numeric hyper unit here?

1.3. Why does the quantum operator include the Planck constant having the dimension of action?

1.4. What is the meaning of the quantum postulate saying values of a quantity are eigenvalues of its operator?

1.5. What is the meaning of *the wave function* ψ – the quantity which the quantum operator is applied to? Why should we consider it as vector of *the Hilbert space* – a special space over the complex number field?

1.6. Why does the wave function require an *interpretation*, whereas other physical quantities do not need any? They signify what they are intended to signify.

2. The current concept is that existing interactions can be reduced to interactions of a handful of elementary particles. We will call such particles *fundamental*. They fall into two groups of particles – leptons and quarks. A remarkable symmetry takes place within each group. Each group is divided into three subgroups (*generations*), two particles (arbitrarily called the *top* and *bottom*) in each.

Hence the UIT should explain the following.

2.1. How do the wave functions of each fundamental particle differ from each other?

2.2. Why are there two particle groups – leptons and quarks?

2.3. Why are there exactly three generations of particles?

2.4. Why does each generation contain exactly two particles? For example, the first lepton generation includes electron e and electronic neutrino ν_e.

Furthermore, it has been discovered that each quark type (*flavor*) exists in three modifications denoted with a color – red, yellow or blue.

Therefore similar problems arise.

2.5. Why are there exactly three quark colors?

2.6. What is the difference between quark wave functions for the same flavor but different colors?

Moreover, each of the above-mentioned fundamental particles has an *antiparticle*.

Hence,

2.7. The UIT should contain concepts and operations related to an antiparticle as well as to a particle.

3. Particle physics is pinning its hopes on the penetration into the unknown physical world of *the supersymmetry*. In this conception, a massive superpartner following opposite statistics corresponds to every fundamental particle. Thus bosons as superparticles correspond to fermions.

It follows that

3.1. The UIT should include the supersymmetry in a natural manner.

The following questions remain unanswered.

3.2. Is the correspondence between particles and superparticles a one-to-one correspondence? For example, is there a superpartner for neutrino?

3.3. Are there quantum phenomena in the world of superparticles?

4. The spacetime of special relativity is the arena where interactions occur. These spacetime transformations that leave interaction processes invariant are defined as the Poincaré group. The Poincaré group includes spacetime shifts, geometrical turns, and Lorentz transformations. The speed of light is the Poincaré group invariant.

This brings up a question: would we construct the Poincaré group generalization by using the UIT?

For example, this question can be initiated by the following. A free light particle moves in accordance with the equation

$$c^2dt^2 - dx^2 = 0.$$

Here x is the coordinate, t is the time, c is the light velocity.

Obviously this equation does not describe light particle kinematics in the emission process. Therefore the above relation should be modified.

Let us decompose the problem into two questions.

4.1. Should we generalize the spacetime of special relativity going towards the UIT?

4.2. Should we generalize the group of geometric rotations and Lorentz transformations?

On the spacetime generalization a question arises:

4.3. Do the quantum phenomena extend over this generalized spacetime?

5. *An inner symmetry group* corresponds to each kind of interaction. It transforms the wave function coordinates belonging to *its inner space*. In addition to these coordinates, the wave function holds coordinates transformed by the Poincaré group. An elementary particle spin is explained by the presence of such coordinates. Let us name the specified coordinate space as *the outer space*. In these terms, *the outer symmetry group* is the Poincaré group.

Hence the following questions arise.

5.1. What do inner spaces of interactions mean?

5.2. What is the meaning of inner symmetry groups?

5.3. Is there any relation between the outer and inner spaces?

5.4. Should we search for a superspace containing the outer and inner spaces?

5.5. Is there any relation between inner spaces of various interactions?

5.6. Should we construct a superspace containing inner spaces of various interactions, and, respectively, an inner symmetry group for all interactions?

6. In the present view, interaction of fundamental particles is performed through *a field*. Exactly, interaction of a particle A with a particle B comes to interaction of the particle B with a field whose *source* is the particle A.

Field kinds correspond to interaction classes. The field quanta are *intermediate particles*. Fundamental and intermediate particles are often mentioned together when enumerating. However, the essential difference between particles is veiled: intermediate particles serve as an interaction agent between fundamental particles and themselves.

It follows that

6.1. The UIT paradigm should reflect the service role of intermediate particles.

7. The UIT should answer the questions concerning interaction classes:

7.1. Why are there *four* interactions in nature: gravitational, weak, electromagnetic, and strong?

7.2. Why is the set of fundamental particles decreased from gravitational to strong interaction? Whereas all fundamental particles are involved in gravitational interaction, the strong interaction is provided only with hadrons.

7.3. Why do interactions differ in their power?

8. The gravitation theory stands apart from the electroweak and strong interaction theories.
It is unclear today:

8.1. Should General relativity represent gravitation in the UIT?

8.2. Which group can be considered as a gravitational group of inner symmetry?

Here we finish the questions for the UIT. Now we propound our answers in the context of our UIT variant and discuss its problems.

III. OUR VARIANT OF UIT: ANSWERS

1. The first set of questions receives answers after the following generalizations.

- Initially, the spacetime is provided with algebraic structure. The vector product is introduced, through which the spacetime becomes an algebra, and specifically *a tensor algebra*.

  With respect to *covariant* basis vectors E^I, the multiplication (composition) rule can be represented as follows

  $$E^I \circ E^K = C^{IK}{}_L \cdot E^L\,, \tag{2}$$

  where $C^{IK}{}_L$ are structure constants (or matrices) of the algebra.

  For *regular (adjoint) representation*, basis vectors correspond to structure matrices

  $$E^I \sim C^{IK}{}_L\,.$$

  The structure matrix number I is an index of the basis vector which is represented by this matrix.

  Hence the generalized *nabla operator* is:

  $$\nabla = E^I \cdot \frac{\partial}{\partial x^I} \sim C^{IK}{}_L \cdot \frac{\partial}{\partial x^I}\,, \tag{3}$$

  where x^I are generalized spacetime vector coordinates.

- The scalar action is generalized into a vector quantity; moreover it is supposed that the set of action vectors S also constitutes a tensor algebra.

  The vector product in this algebra will be written as

  $$S = \frac{1}{S^0}\, S_1 \circ S_2\,. \tag{4}$$

  S^0 is a constant which has the dimension of action, and matches the dimensions of the right and left equation sides. In a particular case this quantity is supposed to be equal to the Planck constant

  $$S^0 = \hbar\,. \tag{5}$$

1.1. In a simplest case of the so called *contracted representation*, expressions (3), (4), (5) result in classic quantum operators of physical quantities, and particularly the momentum operator (1).

1.2. It follows from the product rule of basis vectors (2) that structure matrices include matrix blocks 2×2:

$$\begin{array}{|c|c|}\hline & 1 \\ \hline -1 & \\ \hline\end{array}\,.$$

They may be identified with the imaginary unit i, taking into account that

$$i^2 = -\mathit{1}\,,$$

where a unit matrix is denoted by $\mathit{1}$.

$$\begin{array}{|c|c|}\hline 1 & \\ \hline & 1 \\ \hline\end{array}\,.$$

The imaginary unit thus appears in quantum theory as a result of the algebraic composition law on the spacetime and the action space.

Note that structure matrices include also the following blocks:

$$\begin{array}{|c|c|}\hline & 1 \\ \hline 1 & \\ \hline\end{array}\,, \qquad \begin{array}{|c|c|}\hline -1 & \\ \hline & 1 \\ \hline\end{array}\,.$$

Identifying these matrices with numbers a and b, respectively, we should get hypernumbers with unities

$$\{\mathit{1}, a, b, i\}$$

and the product rule

$$a^2 = b^2 = \mathit{1}\,, \qquad i^2 = -\mathit{1}\,, \qquad a\,b = -b\,a = i\,,$$
$$a\,i = -i\,a = b\,, \qquad i\,b = -b\,i = a\,.$$

1.3. It seems unreal that the Planck constant transforms to the quantum operator from the composition law (4). It would naturally define the quantum operator as the nabla operator (3) and assign the Planck constant to the composition law (4).

1.4. We differentiate the law composition (4) twice and evaluate $d_2 d_1 S$. Here d_1 and d_2 are differentials in the direction of vectors S_1 and S_2, respectively. We obtain

$$d_2 d_1 S = \frac{1}{\hbar}\, d_1 S \circ d_2 S\,. \tag{6}$$

This expression is *a structure equation* of action algebra in vector form.

Let us introduce a notation

$$\psi = d_1 S$$

and $d \sim d_2$. Then the structure equation (6) becomes

$$d\psi = \frac{1}{\hbar}\,\psi \circ dS\,. \tag{7}$$

The equation now is in the form of an eigenvalue problem for the differential operator d.

We have the following result: The structure equation of the action algebra may be reduced to an eigenvalue problem for the differential operator d. Therefore, the quantum postulate is nothing other than the action algebra composition law in the differential form.

1.5. From Eq. (7) it follows that the differential of an action vector (the quantity ψ) should be identified as the wave function being introduced in the quantum theory.

The Hilbert space should be considered as a tensor algebra of action.

1.6. The necessity to interpret the wave function is a consequence of an underdeveloped quantum theory basis, and illustrates the absence of a quantum phenomena explanation. The above considerations explain quantum phenomena by representing the action as an algebraic quantity (by algebraic structure of action space).

2. The second set of problems is solved by a detailed analysis of the tensor action algebra. It is convenient to illustrate the tensor algebra division into subalgebras (graphically by means of a diagram named *the Young tree* (Fig. 1). Here, figures signify tensors included in the wave function. The number of cells in the figure signifies the tensor rank. Its own tensor symmetry by the permutation of indices corresponds to each figure. An antisymmetric combination of indices corresponds to the vertical (disposition) arrangement of cells but a symmetrical combination of indices corresponds to the horizontal arrangement. A line connecting a unicellular figure with one of the four-cellular figures (*a trunk*), corresponds to a subalgebra of the tensor algebra.

Thus, each subalgebra includes tensors from the first to fourth rank with a certain symmetry. It will be noted that to the leftmost trunk corresponds a subalgebra containing only antisymmetric tensors. This is the Clifford algebra C_4. Restriction by antisymmetric tensors from one to three ranks, we will achieve the Clifford algebra C_3.

2.1. The fundamental particles are in correspondence to subalgebras of the tensor algebra. More precisely the wave functions of fundamental particles are subalgebra vectors of the tensor algebra, and differ from each other in their symmetries of tensors which constitute the algebra. For example, the electron wave function is the vector of the Clifford algebra C_3, and the lepton wave function of one generation is the vector of the Clifford algebra C_4.

All the subalgebras of the tensor algebra at the second rank tensor level and respectively the fundamental particles are divided into two classes. Wave functions of the first class particles contain an *antisymmetric* tensor of the second rank and wave functions of the second class particles contain a *symmetric* tensor of the second rank. The division of fundamental particles into two classes corresponds to such wave function classification: particles with the spin 1/2 are fermions and particles with the spin 0 are bosons. Leaving aside the last particles for a while we shall turn our attention to the fermions. The left branch of the Young tree illustrates subalgebras relating to fundamental fermions.

2.2. The fermion subalgebras at the third rank tensor level are divided into two groups. One of them includes an antisymmetric tensor of the third rank and represents the Clifford algebra whose vectors, according to Dirac, describe leptons. It is natural to relate the second group of subalgebras to quarks.

Therefore, we may, within the particle classification by subalgebras of the tensor algebra, explain the division of fundamental particles with spin into two sets (groups) leptons and quarks, and also interpret algebraically such properties as lepton and baryon charges.

2.3. Let us now turn to the question: Why are there three generations of particles? We should notice that for each subalgebra, the wave functions may differ in *the order* of their geometric space basis vectors. So, the wave function of the first generation particles corresponds to the order of indices 123. Just for this index order the Young tree (Fig. 1) is shown. Wave functions of the second and third generation particles correspond to index orders 312 and 231, respectively.

Thus, wave functions of several lepton generations differ from each other by a cyclic permutation of three basis vectors of the generating geometric space. This rule also applies to wave functions of several quark generations. The difference in mass of several generation particles testifies that the geometrical space of fundamental particles is probably inhomogeneous. The reason for the heterogeneity remains vague.

2.4. Let us now find out why every lepton and quark generation contains two particles.

The quantum equations for free leptons are divided into two independent systems of equations. They may be related to the bottom lepton and its neutrino. Hence, the subalgebra for each generation of leptons relates to *the lepton-neutrino pair*.

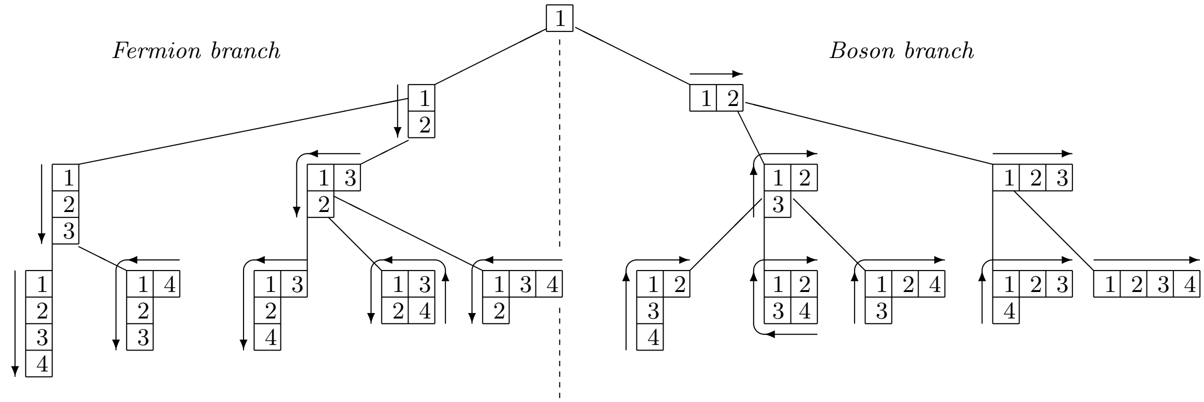

FIG. 1. Four-level Young tree of the first generation.

The quantum equations for free quarks are also divided into two independent systems of equations. They may be related to the top and bottom quarks. Hence, the subalgebra for each generation of quarks relates to *the quark pair*.

The wave functions of free leptons and quarks are divided into two components – the right and the left. This division allows us to describe electoweak interaction involving leptons and quarks.

2.5. Tensors of the fourth rank are constructed by means of the timelike basis vector. The quark algebra at *a relativistic level* divides into three subalgebras by three symmetries of the four-rank tensor. Each of these subalgebras can be associated with a quark of a certain *color* – red, green or blue. So, in the framework of our particle classification, the quark color obtains an algebraic interpretation.

2.6. Hence it follows that wave functions of quarks with the same flavor and different color vary in symmetries of four-rank tensors included in these functions.

Let us point out an interesting consequence of the algebraic classification of fundamental particles. It concerns leptons. Like quark algebra, the lepton algebra at the relativistic level is divided into two subalgebras. It follows that there are two types of leptons. It is convenient them to designate by color as *white and black leptons*. Then it should be concluded that white and black leptons are related by short-range forces of a color attraction exceeding the Coulomb force (for charged leptons).

In particular, these forces are revealed to form electronic pairs in such phenomena as

- the filling of atomic orbits by electrons,
- the covalent bond,
- the formation of crystal lattice,
- the superconductivity.

2.7. To describe an antiparticle the contravariant action algebra is substituted for the covariant one. Structure matrices in the real presentation are converted to transposed matrices, and structure matrices in the complex presentation are converted to hermitian-conjugated ones. In particular, charge matrices change the sign.

3. Let us now consider problems associated with the supersymmetry. We will choose *the boson branch* from the Young tree now. It is shown in Fig. 1 that the subalgebra of the boson branch corresponds to each subalgebra of the fermion branch. That is, each particle with spin 0 corresponds to each particle with spin 1/2.

3.1. So, our algebraic approach results naturally in the concept of supersymmetry. We call supersymmetric to leptons particles *a leptino*, and call supersymmetric to quark particles *a quarkino*.

Note that this considered supersymmetry concept differs from the traditional one. Among the superparticles there are particles neither with integer nor with fractional spin. The superparticle spin should be considered as equal to zero. More precisely a superparticle has no such property as the spin, and a new dynamic property, in a certain sense, symmetric to spin, takes its place. We call the property *an inertia*. In the superparticle wave function the inertia is represented as a second rank *symmetric* tensor. Fundamental superparticles do not participate in the interactions where fundamental particles participate except for the gravitation.

In the Young tree (Fig. 1) we specify the direction of increasing fundamental particle mass from leptons to quarks, i.e. from left to right. Then it should be considered that the quarkino is heavier than the quark, and the leptino is the heaviest particle. Superparticles do not participate in electromagnetic interactions. This suggests that superparticles probably constitute so-called *dark matter*, and superfields form *dark energy*.

3.2. At the relativistic level the quarkino algebra is divided into three subalgebras, the leptino algebra is divided into two subalgebras. We relate these three subalgebras to quarkino of three colors – red, green and blue. We also have to suppose that the leptino is of two colors – black and white. The equations for free superparticles can not be divided into two equation systems. Thus, there is a single superparticle with the top and the bottom components mixed among themselves (for example, there are not superanalogues of electronic neutrino and electron).

3.3. The algebraic structure of superparticle action vectors determines the quantum postulates and quantum phenomenon theory of these particles. In superparticles quantum theory, the unity previously designated as a plays the role of the imaginary unit. Generally, there is no reason to consider that the superparticles quantum phenomena use the Plank constant, rather than some other constant quantity with dimension of action.

4. Our UIT includes a generalization of the Poincaré group. Analysis of the Dirac equation shows that every one of the fundamental particles has its proper spacetime. This generalizes the spacetime of special relativity.

4.1. The proper spacetime of the fundamental particle is a subalgebra of the tensor algebra whose space generator is the spacetime of special relativity. In particular, the generalized spacetime of white leptons is the Clifford algebra C_4. This generalization is based on involving additional coordinates as independent spacetime variables.

Besides

- x^a geometric coordinates and
- t time coordinate,

new coordinates are used:

- s^{ab} coordinates of area (or of rotation angle),
- v^a speed coordinates,
- V^{abc} coordinates of volume (or of solid angle),
- ω^{ab} angular velocity coordinates,
- Ω^{abc} solid angular velocity coordinates,
- s the length of generalized spacetime vector.

Where indices a, b, c take on values 1, 2, 3.

Additional momentum components correspond to the additional coordinates. Besides

- 3-dimensional momentum p_a and
- energy

$$p_4 = \frac{E}{c}\,,$$

new momentum components are used:

- the 3-dimensional angular momentum

$$p_{ab} = \frac{M_{ab}}{R}\,,$$

- the 3-dimensional force

$$p_{a4} = T\,f_a\,,$$

- the 3-dimensional solid angular momentum

$$p_{abc} = \frac{M_{abc}}{R}\,,$$

- the 3-dimensional angular torque

$$p_{ab4} = \frac{F_{ab}}{c}\,,$$

- the solid angular torque

$$p_{abc4} = \frac{F_{abc}}{c}\,,$$

- the rest momentum p_0.

Where R is the fundamental constant of length, T is the fundamental constant of time.

Standard 4-dimensional special relativity is generalized for the fundamental particle spacetime. For example, the motion of the light particle satisfies the equation

$$c^2dt^2 - dx^2 - \frac{c^2}{A^2}dv^2 = 0.$$

Where v is the speed of the light particle, A is the fundamental constant, so-called *maximum acceleration.*

4.2. The geometrical rotations and Lorentz transformations from the Poincaré group are generalized to rotations about all basis vectors in the generalized spacetime.

Since a fundamental antiparticle corresponds to each of the fundamental particles, the conjugate transformations of antiparticle spacetime can be included into the invariant transformations. The above transformations are combined into *a general kinematic algebra* which generalizes the classical invariant transformation group – the Poincaré group.

4.3. In contrast with the spacetime of special relativity, the proper spacetime of a fundamental particle is an algebra. Therefore this spacetime is quantized. Initial quantum postulates are the structure equations of the corresponding algebra.

5. The fifth set of questions will receive answers after the following considerations. The unit sphere of the action algebra (the unit is the Planck constant) divides the algebra definition domain into two parts – inside and outside the sphere. We will consider that the product inside the sphere is *right*:

$$S = S_1 \circ S_2 \,.$$

Then the product outside the sphere is *left*:

$$(S)^{-1} = (S_2)^{-1} \circ (S_1)^{-1} \,.$$

5.1. The action algebra domain inside the unit sphere is identified as *an inner space* in the sense as it is understood in elementary particle physics. Such understanding of the inner space is based on the fact that the regular representation of basis action algebra vectors with *right* product leads to the charge matrices.

5.2. The inner symmetry groups acquire the meaning of action algebra vector rotations inside the unit sphere. For example, the symmetry group of electromagnetic interactions is a group of rotations about the basis vector e_{21}. It is isomorphic to the group $U(1)$. The weak interactions group is a group of rotations around the basis vectors e_4 e_{123}, e_{1324}. It is isomorphic to the group $SU(2)$.

5.3. The action algebra domain outside the unit sphere is identified as *the outer space* in the same sense as in Section II.5. Such understanding of the outer space is based on the fact that the regular representation of basis action algebra vectors with *left* product leads to space-time matrices, in particular, to the Dirac matrices.

5.4. The space enveloping the inner and outer spaces is the action algebra in the whole definition domain. The generalized metric tensor is used to change the outer contravariant space to the inner covariant space.

5.5. The inner spaces of various interactions are combined in the inner space of the particle being involved in these interactions.

5.6. The question whether we should we search for a space encompassing the inner spaces of various interaction types and, respectively, an inner symmetry group encompassing inner symmetry groups of different interaction types has been answered essentially in items 5.2, 5.5.

6. The sixth set of questions receives answers after the following considerations.

6.1. The action of an intermediate particle is a linear transformation operator applied to the action vectors of fundamental particles. The action of an intermediate antiparticle is a linear transformation operator applied to the action vectors of fundamental antiparticles. The action vectors of an intermediate particle compose an algebra. The action vectors of an intermediate antiparticle also compose an algebra.

Let e_I be basis vectors of the fundamental particle action algebra, and $I^K{}_L$ be basis vectors of the intermediate particle action algebra.

Then, the interaction of fundamental and intermediate particles follows the algebraic composition law

$$e_I \circ I^K{}_L = \delta^K{}_I \cdot e_L \,.$$

Where $\delta^K{}_I$ is the Kronecker symbol.

Some considerations compel us to postulate the existence of intermediate particles of *a second kind*. Therefore the above intermediate particles are named as ones of *the first kind*.

Let $J^{KL}{}_M$ be the action algebra basis vectors of an intermediate participle of the second kind. Then the interaction between fundamental and intermediate particles of the second kind results from the algebraic composition law.

$$e_I \circ J^{KL}{}_M = \delta^K{}_I \cdot I^L{}_M \,.$$

It means that the interaction of fundamental and second kind intermediate particles produce first kind intermediate particles.

7. The seventh set of problems does not have in essence any solutions. Only some considerations may be formulated.

7.1. A general group of linear transformations acts on the inner space. The group is an inner symmetry group of the general form. Its subgroups define the interaction types.

7.2. Suppose, the basis rotational vector e_α specifies an inner symmetry group. The index α indicates a tensor component, and, in particular, the rank of the tensor included in the fundamental particle action. For example, the basis vector e_{12} relates to the second rank tensor and the basis vector e_4 relates to the first rank tensor. It seems, the general law is as follows: the higher the tensor rank, the narrower the set of particles involved in the corresponding interaction.

7.3. The higher the tensor rank, the "stronger" the interaction.

8. The isolation of the general relativity theory as a theory of the gravitational interaction arises from the fact that its mathematical basis – the Riemannian space, is not a vector space supplied with a scalar product. And this is unacceptable from a physical standpoint. The matter is that the multiplication of a vector by a number and the scalar product of vectors are the mathematical equivalent for the procedure of vector quantity measurement. Hence,

8.1. The attempt to unify the gravitational interaction with electroweak and strong interactions should be accompanied by reformulation of the gravitation theory.

8.2. It is necessary to find an inner symmetry group of gravitation.

Finally, I do hope that this article will attract the attention of researchers whose train of thought is compatible with the presented considerations.

ACKNOWLEDGMENTS

I also thank Dr. A. I. Tsirlin and N. Jones for their help in preparing the article and important comments.

[1] A. A. Ketsaris, *Foundations of mathematical physics*, Moscow, Association of independent publishers, 1997, 280 pp., in Russian.

[2] A. A. Ketsaris, *Accelerated motion and special relativity transformations*, arXiv:physics/9907037

[3] A. A. Ketsaris, *Turns and special relativity transformations*, arXiv:physics/9909026

[4] A. A. Ketsaris, *Generalized equation of relativistic quantum mechanics*, arXiv:quant-ph/9909053

[5] A. A. Ketsaris, *Generalized equation of relativistic quantum mechanics in a gauge field*, arXiv:quant-ph/9909087

[6] A. A. Ketsaris, Lectures on Unified interactions theory, `http://toe-physics.org/en/lectures.htm`, 2003–2012, in Russian.